

**Gregory S. Amacher\* – Erkki Koskela\*\*- Markku Ollikainen\*\*\***

**OPTIMAL ROYALTY DESIGN AND ILLEGAL LOGGING  
UNDER ALTERNATIVE PENALTY SCHEMES\*\*\*\***

Department of Economics, University of Helsinki.

Discussion papers No 570:2003.

ISSN 1459-3696

ISBN 952-10-1219-6

June 16, 2003.

- \* Department of Forestry, College of Natural Resources, 307 Cheatham Hall, Virginia Polytechnic Institute and State University, Blacksburg, VA 24060, USA. Email: [gamacher@vt.edu](mailto:gamacher@vt.edu).
- \*\* Department of Economics, P.O. Box 54 (Unioninkatu 37), FIN-00014 University of Helsinki, Finland, and the Research Department of the Bank of Finland, P.O. Box 160, 00101 Helsinki, Finland. Email: [erkki.koskela@helsinki.fi](mailto:erkki.koskela@helsinki.fi).
- \*\*\* Department of Economics and Management, P.O. Box 27, FIN-00014 University of Helsinki, Finland. Email: [markku.ollikainen@helsinki.fi](mailto:markku.ollikainen@helsinki.fi).
- \*\*\*\* This paper is a part of the project “*Studies in Environmental and Resource Economics*” financed by the Academy of Finland. The funding is gratefully acknowledged. Koskela thanks the *Research Unit on Economic Structures and Growth* (RUESG) at the University of Helsinki for financial support and the Bank of Finland for hospitality. Ollikainen thanks the Academy of Finland for financial support.

**Abstract:**

We study forest royalty and enforcement design in the context where concessions are allocated by a government and illegal logging is present. The government is motivated by royalty revenue collections, by public goods provided from forests, and by the welfare of concessionaires. When harvesters are risk neutral, the optimal policy mix depends on the presence of amenities but not on the penalty scheme. For risk-averse harvesters the results differ. When the penalty is assessed on undeclared income, a royalty exemption is not optimal, but when penalties are levied on evaded royalty payments, the optimal royalty system may be progressive or regressive depending on the size of the government's budget revenue requirement. Auditing is optimal under both schemes. Accounting for amenities implies a higher optimal royalty rate, lower progression and increased auditing.

**Keywords:** illegal logging, royalty design, penalty schemes.

**JEL classification:** D81, H26, Q21, Q23.

## 1. Introduction

In most of the countries with tropical forests, concessions are the primary means by which government-owned native forests are harvested (Walker and Smith 1993, Gray 2000). Often governments view concessions as a means for providing stable wood flows to domestic forest industry (Verissimo et al. 2002). Royalties charged for harvesting concessions are also popular means for tropical country governments to generate revenue collections, and royalty revenues provide large sources of funds to many developing country treasuries (Gray 2000, Amacher et al. 2001).<sup>1</sup>

There has been considerable debate concerning the proper design of royalty instruments in concession economies. Two problems in countries with distant forest concessions and inefficient centrally-located governments are illegal logging (a main component of tropical deforestation) and the inability or unwillingness of governments to enforce and penalize such acts (e.g., see Clarke et al 1993 for a discussion of lack of monitoring of harvest activities in developing countries, and Palmer 2000). In practice, large amounts of harvests are not reported to authorities, and harvesting often occurs well beyond boundaries specified in concession contracts. Often governments cannot perfectly detect illegal activities, nor do they always seek to collect penalties even when illegal logging is detected. Illegal logging and underreporting of harvest income undermine a government's ability to collect revenue. Both activities may also reduce public goods provided from native forests and increase net rents to harvesters (ITTO 2002, Repetto and Gillis 1988).

Sometimes it has been suggested that revenue collection motivations of governments might run counter to the goals of providing amenities from preserving tropical forests (Palmer 2000, Amacher 1999, Poore 1993). It has also been argued that current royalty rates are far too low, providing little incentive to curb excessive harvesting (Gray 2000, Vincent 1990, Merry et al. 1998). Moreover, poorly-designed royalty systems have been linked to incentives for illegal logging (Palmer 2000).

From this literature, one might imagine that increasing royalty rates is the simple solution for increasing government revenues and slowing deforestation.

However, one has not considered these issues when there is an incentive for harvesters to cheat and evade royalty payments or to illegally log in a concession. The probability that harvesters are caught by the concession-awarding government is also a factor in their illegal activity choices. All of this means that the actual design of royalties in practice remains an interesting problem with no clear solution.<sup>2</sup>

In this paper we study forest royalty and enforcement design, in a context where concessions are allocated by a government and harvesters have incentives to illegally log. Illegal logging is modeled, realistically, as excessive harvesting beyond a level permitted by the concession agreement. Illegal logging is not detectable by the government unless costly enforcement is employed. With enforcement, the government can detect illegal logging only with some probability less than one. Thus, detection occurs through auditing, and the cost of this auditing represents the enforcement efficiency of the government. Should the government detect illegal logging, one of two penalty schemes used in practice is assumed applied: a fine collected on undeclared (unreported) harvest income, or a fine collected on evaded (non-paid) royalty payments.

Our paper is the first to consider royalty and auditing design under the possibility of illegal activity among harvesters. It is related to work in public finance on optimal taxation design when tax evasion is present, and where governments must enforce tax systems because of illegal behavior by taxpayers. In addition to our different context, the presence of amenities, closely tied to the forestry problem, creates important differences between our work and this literature.<sup>3</sup>

Within our new framework we evaluate two questions, first, how do incentives for harvesters to illegally log depend on government auditing effort and royalty choices,

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<sup>1</sup> In fact, Brazil is about to open up over 70 million hectares of public Amazon forests for concessions, with a cited purpose being the generation of government revenues through fees attached to the concessions and sustainable forest harvesting for timber concerns (MMA 2001).

<sup>2</sup> Royalties and logging behavior have been studied recently. Vincent and Boscolo (2000) analyze the impact of royalties on use of improved logging practices, such as reduced impact logging and diameter limits. Their results are simulation-based and royalty/auditing choice and the possibility of illegal logging and penalties are not considered (they do acknowledge the potential difficulties of monitoring logging, however). Clarke et al (1993) examine the role of penalty schemes and optimal dynamic enforcement expenditures for an open access forest situation by considering a Nash game between authorities and forest poachers, where detection of illegal logging leads to prosecution. Walker and Smith (1993) model noncompliance of loggers facing a given concessions contract, and they formulate a type of auditing procedure that can limit noncompliance. But the choices of royalty and auditing policies from the perspective of a government, under revenue constraints and illegal logging incentives for harvesters, are not examined in this collection of work.

among other parameters?, and, second, how should a government choose royalties and audit frequencies when trying to improve harvester welfare and facing revenue needs, but also attempting (in some cases) to protect amenities from preserving forest stocks. We allow for the notion of a government motivated by royalty revenue collections, possibly interested in public goods provided by the forests, and responsive to welfare of concessionaires.

As we will demonstrate, illegal logging and government revenue constraints make the design of royalties more complicated than what is often assumed. Solutions to deforestation are not as simple as just raising royalty fees or shifting more resources toward auditing of harvesters. Each action has both welfare and government revenue implications. Our results provide some guidance in the current debate over application of royalties to control deforestation by ameliorating illegal logging. We show that royalties and auditing for illegal logging should be designed as a system and not considered, as they often are, as separate instruments. Perhaps most importantly, we establish the importance of risk attitudes of harvesters, amenities associated with reductions in illegal harvesting, and penalty schemes for the optimal design of policies. Finally, we show that governments may never find it optimal to either eliminate illegal logging or enforce penalties under some conditions.

The rest of the paper is organized as follows. In section 2 we present a simple model how illegal logging depends on penalty schemes, audit strategies and royalties, and risk attitudes of the harvester. In section 3 we study optimal royalty and auditing design for a risk neutral harvester, for both penalty schemes, and cases where amenities are either present or absent. We extend this analysis in section 4 for the case of risk-averse harvesters. The last section contains our conclusions.

## 2. A Basic Model of Illegal Logging

Consider a concessionaire (i.e., harvester) who receives a permit to harvest a concession of known size. This harvester's objective is to maximize their rents from harvesting. Suppose that the harvester receives a right to harvest  $Q = \bar{Q}$ . Governments

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<sup>3</sup> For surveys on the public finance literature on tax evasion, see e.g. Cowell (1987) and Myles (1995).

normally impose royalties, or fees, for the right to harvest the concession. We consider a popular and common form of royalty applied to harvesting, i.e., a fee assessed against the value permitted for harvest,  $tq\bar{Q}$ , where  $t$  is the royalty rate and  $q$  is the logging timber price.<sup>4</sup> If the harvester is honest, then he harvests exactly the amount permitted for the concession, and nothing more. The profit function for an honest harvester is written,

$$\pi = q\bar{Q}(1-t) - c(\bar{Q}), \quad (1)$$

where  $c(\bar{Q})$  is a convex cost of harvesting, i.e.  $c'(\bar{Q}), c''(\bar{Q}) > 0$ .<sup>5</sup>

If the harvester's rent is positive at  $\bar{Q}$ , i.e., if  $q(1-t) - c'(\bar{Q}) > 0$ , then there is an inherent incentive for illegal logging. Given that forest concessions are often distant from the auditing authority, this creates a problem of moral hazard. Assuming an imperfect detection of harvesting behavior by the government, as it is in all developing countries with concessions, the harvester engages in illegal logging. Denote illegal logging by  $X$  and let it be defined as excessive harvesting, expressed by the difference between actual logging  $Q$  and permitted logging  $\bar{Q}$ , i.e.,  $X \equiv (Q - \bar{Q})$ .<sup>6</sup>

Let  $p(p \leq 1)$  denote the probability of illegal logging detected by the government. The government is not assumed to audit the harvester one hundred percent of the time, and so the frequency of auditing (i.e., the probability of detecting illegal logging) is a measure of the intensity with which the government monitors and detects illegal activities. We consider two types of *penalties* imposed by the government should

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<sup>4</sup> For examples and discussions of this type of royalty in both the concessions literature and practice, see Gray 2000. The royalty considered here is similar in spirit to an 'area royalty' if we think of a concession in terms of total volume in a given area. In this case, some quantity of harvest volume is exempted from the tax payment made by the harvester. Many concessions have been applied in this manner.

<sup>5</sup> In what follows, derivatives of a function with one argument is denoted with primes, while partial derivatives of functions with more than one argument are denoted by subscripts.

<sup>6</sup> 'Excessive (illegal) harvesting' could be thought of as harvesting too much area, or as removal of too much volume. Timber trespass is a common form of the former, while high grading is a common form of the latter. With high grading, either too much volume is removed, or too much volume is removed from high valued species groups. We can consider the concession above in terms of species groups if we think of  $\bar{Q}$  as a vector of species harvest allowances.

cheating be detected.<sup>7</sup> The first penalty is a fine assessed against the concessionaire's undeclared profits. These are profits, not reported to the government, that the concessionaire earns from illegal logging. The concessionaire's precise profits in this penalty scheme would now depend on whether illegal logging is detected. If illegal logging is not detected, then the harvester's profits are defined as,

$$Y^u = [qQ^u - tq\bar{Q} - c(Q^u) + I] \quad (2)$$

and if it is detected, then profits include the penalty,

$$Z^u = [Y^u - fqX^u], \quad (3)$$

where the 'u' superscript indicates the case where penalties are assessed on undeclared (evaded) profits,  $f$  is the fine rate,  $X^u$  is illegal harvesting under this penalty, and the variable  $I$  represents non-logging income (we make use of this later).

Another penalty system we will consider is one where the penalty rate is levied on the actual evaded royalty payments. These payments amount to royalties the government would have collected from the harvest if the concessionaire truthfully reported the harvest level to the government. Now the concessionaire's profit is written,

$$Y = [qQ - tq\bar{Q} - c(Q) + I], \quad (4)$$

if illegal logging is not detected with probability  $(1 - p)$ , and

$$Z = [Y - ftqX], \quad (5)$$

if illegal logging is detected with probability  $p$ , where  $X$  represents illegal logging under this penalty.

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<sup>7</sup> The fine structures we use here are common in government documents for concessions programs (Gray 2000) and have support in other literature on tax evasion (e.g. see Yitzhaki 1974 for penalties levied against evaded taxes and Allingham and Sandmo 1972 for penalties levied on undeclared incomes).

Given the positive probability of being caught, the precise incentive to illegally log depends on the harvester's risk preference. We will allow for both neutrality and aversion towards risk.

## 2.1. Incentives for Illegal Logging: Risk Neutral Harvesters

The risk-neutral harvester maximizes expected profits, defined under the two penalty schemes as,

$$\underset{Q}{\text{Max}} E\pi = qQ^u - c(Q^u) - tq\bar{Q} + I - pqfX^u, \text{ and} \quad (6)$$

$$\underset{Q}{\text{Max}} E\pi = qQ - c(Q) - tq\bar{Q} + I - ptqfX \quad (7)$$

First-order conditions for harvesting in each case are, respectively,

$$Q^u : q - c'(Q^u) - pqf = 0, \text{ and} \quad (8)$$

$$Q : q - c'(Q) - ptqf = 0 \quad (9)$$

Both conditions show that optimal logging intensity is defined by the equality of marginal revenue ( $q$ ) and expected marginal cost, which consists of the harvest cost plus the expected fine payment ( $c'(Q^u) + pqf$  or  $c'(Q) + ptqf$ ).

It is easy to show that illegal logging depends on exogenous parameters as follows.

$$X^u = X^u(\underset{+}{q}, \underset{-}{p}, \underset{-}{f}, \underset{0}{t}, \underset{0}{I}) \quad \text{and} \quad X = X(\underset{+}{q}, \underset{-}{p}, \underset{-}{f}, \underset{-}{t}, \underset{0}{I}) \quad (10)$$

The effects of  $q, f, p$  and  $I$  on illegal logging are similar in both penalty cases. A higher timber price increases illegal logging, while a higher probability of detection and penalty fine reduce it. Changes in non-logging income will have no effect.

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Interestingly, the effect of the royalty rate  $t$  on illegal logging depends on the penalty scheme. When the penalty is based on undeclared income, the royalty rate has no effect on actual logging. However, higher royalty rates decrease illegal logging when the penalty if caught is levied on evaded royalty payments. This difference arises because, in this case, increases in the royalty rate introduce a negative substitution effect on the actual logging decision.

## 2.2 Incentives for Illegal Logging: Risk-Averse Harvesters

A risk-averse harvester's behavior will differ from that of a risk-neutral harvester. When the penalty is levied on undeclared income, the concessionaire maximizes the following expected utility function,

$$\underset{Q^u}{\text{Max}} EU(\pi) = (1-p)u(Y^u) + pu(Z^u), \quad (11)$$

where  $Y^u = [qQ^u - tq\bar{Q} - c(Q^u) + I]$  and  $Z^u = [Y^u - fqX^u]$ . The first-order condition for the optimal logging is,

$$EU_{Q^u} = (1-p)u'(Y^u)a + pu'(Z^u)b = 0, \quad (12)$$

where  $a = q - c'(Q^u) > 0$  and  $b = q - c'(Q^u) - fq < 0$ . The second-order condition can be expressed as,

$$EU_{Q^u Q^u} = (1-p)u'(Y^u)a[-A(Y^u)a + A(Z^u)b] - c''(Q^u)((1-p)u'(Y^u) + pu'(Z^u)) < 0, \quad (13)$$

where  $A(Y^u) = -\frac{u''(Y^u)}{u'(Y^u)}$  and  $A(Z^u) = -\frac{u''(Z^u)}{u'(Z^u)}$  denote the Arrow-Pratt measures of absolute risk-aversion (see Arrow 1974).

An interior solution requires  $EU_{Q^u} |_{Q^u=\bar{Q}} > 0 \Leftrightarrow q - [c'(Q^u) + pfq] > 0$ . Thus for a risk-averse harvester to engage in illegal logging, the timber price must exceed the expected marginal cost of logging, which now includes the marginal cost of harvesting and the expected fine payment.

The comparative statics of harvesting and illegal logging in this case are reported in Appendix 1. We can express the effects of changes in exogenous variables as follows

$$X_f'' < 0 \quad \text{and} \quad X_p'' < 0 \quad (14a)$$

$$X_I'' \geq 0 \quad \text{and} \quad X_t'' = q\bar{Q}X_I'' \leq 0 \quad (14b)$$

Thus, a higher penalty rate and higher probability of detection both decrease illegal logging. Higher wealth increases illegal logging under decreasing absolute risk aversion (DARA), but it has no effect under constant risk-aversion (CARA). This makes sense given that higher wealth of the harvester increases the incentive to capture rents in a risky manner, as long as risk aversion is decreasing in wealth. Positive non-logging income  $I$  can also be interpreted as a lump-sum subsidy given to the harvester. A negative value for  $I$  is equivalent to a lump-sum fee assessed for the concession. With this interpretation, the signs in (14b) are reversed, i.e.  $X_I'' \leq 0$  and  $X_t'' = -q\bar{Q}X_I'' \geq 0$ . Here, a higher royalty rate would induce only an income effect but no substitution effect. Thus, harvesting and illegal logging are inversely related to the royalty rate  $t$  under DARA.

Turning now to the case where the penalty is levied on evaded royalty payments, the concessionaire's expected utility maximization problem becomes,

$$\text{Max}_{\bar{Q}} EU(\pi) = (1-p)u(Y) + pu(Z), \quad (15)$$

where  $Y$  and  $Z$  are as defined earlier. The first- order condition is,

$$EU_{\bar{Q}} = (1-p)u'(Y)\hat{a} + pu'(Z)\hat{b} = 0, \quad (16)$$

where  $\hat{a} = q - c'(Q) > 0$  and  $\hat{b} = q - c'(Q) - tfq < 0$ , and the second-order condition is,

$$EU_{QQ} = (1-p)u'(Y)\hat{a}[-A(Y)\hat{a} + A(Z)\hat{b}] - c''(Q)[(1-p)u'(Y) + pu'(Z)] < 0. \quad (17)$$

The first order condition (16) shows that the harvester's choice of logging equates expected marginal benefits from harvesting to expected marginal costs. An interior solution for illegal logging implies  $EU_{Q|Q=\bar{Q}} > 0 \Leftrightarrow q - [c'(Q) + ptfq] > 0$ , with the interpretation being similar to the earlier case.

The comparative statics of illegal logging for the fine and detection probability, and timber price parameters, are qualitatively the same as when the penalty is assessed on undeclared income. But there is an important difference between penalty schemes concerning the effect of the royalty on harvesting and illegal logging. When penalties are levied against evaded royalties, the royalty rate  $t$  is now distortionary. In addition to having an income effect, it also induces a substitution effect on harvesting. In Appendix 2, we demonstrate that an increase in the royalty rate reduces illegal logging, i.e.,  $X_t < 0$  under DARA, when the penalty is levied on undeclared income, and even under CARA when the penalty is levied on evaded royalty payments. We summarize our comparative statics results in the following,

**Result 1. Illegal logging under risk-aversion and alternative penalty schemes.**

*Illegal logging is inversely related to the fine rate and the probability of detection for both penalty schemes. Illegal logging is inversely related to the royalty rate through an income effect under decreasing absolute risk aversion when the penalty is levied against undeclared income, but through income and substitution effects when the penalty is levied against evaded royalty payments. The latter also holds under constant absolute risk aversion.*

### 3. Optimal design of royalties and auditing schemes – Risk neutrality

We now examine the optimal design of royalties and auditing when the government seeks to raise a given amount of expected revenues and harvesters are risk neutral. The government is assumed to maximize a social welfare function, which includes the profits of the concessionaire and negative externalities caused by illegal logging. In

practical terms, these externalities are associated with deforestation of expanding forest frontier boundaries, or with high grading of existing stocks, or both. Therefore, we parameterize an amenity function that depends on illegal logging by writing a function,  $v(X)$ , with  $v'(X) < 0$ . The derivative indicates there are social costs associated with deforestation caused by illegal logging. The social welfare function can now be written as,

$$SW = E\pi + v(X), \quad (18)$$

The government must expend some type of auditing effort. Let the cost of auditing incurred by the government be an increasing and convex function of the probability of detection,  $c(p)$ , such that  $c'(p) > 0; c''(p) > 0$ . We further assume that  $c'(0) = 0$ .<sup>8</sup> Expected government revenues depend on royalty payments collected, fines collected, and the cost of auditing as a function of the probability of detection. When penalties for illegal logging are levied on undeclared income, government revenues are specified as,

$$R^e = tq\bar{Q} + pfqX^u - c(p). \quad (19)$$

When penalties are levied against the evaded royalty payment then we have,

$$R = tq\bar{Q} + ptfqX - c(p). \quad (20)$$

### 3.1 Penalty charged on undeclared income

We first examine policy design under the assumption that amenities are not important or are ignored by the government. Thus, the government chooses the royalty rate  $t$  and the audit probability  $p$  to maximize the expected indirect profit function  $E\pi^*$  subject to

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<sup>8</sup> The high cost of auditing and enforcement of concessions activities has been cited as one reason why illegal logging is a problem in tropical countries with concessions. Because we define the cost of auditing as a function of the audit probability, it could in principle capture several features of this cost in practice, such as the long distance between the central government and harvesting location, or the inefficiency of the government in undertaking these actions.

$R^e = tq\bar{Q} + pfqX^u - c(p) \geq \bar{R}$ .<sup>9</sup> Writing the Lagrangian function for this problem as  $\Omega^u = E\pi^* + \lambda(R^e - \bar{R})$ , and accounting for the harvester's response, we have the following necessary conditions for the choice of  $t$  and  $p$ :

$$\Omega_t^u = -q\bar{Q} + \lambda q\bar{Q} = 0, \text{ and} \quad (21)$$

$$\Omega_p^u = -fqX^u + \lambda[fqX^u + pfqQ_p^u - c'(p)] = 0. \quad (22)$$

The government chooses a policy to equate marginal revenue collections,  $\lambda q\bar{Q}$ , to marginal costs,  $q\bar{Q}$ , defined in terms of the effects they have on welfare of the concessionaire. The multiplier of the government's revenue constraint is the marginal effect on social welfare of additional revenue collected, i.e. the marginal cost of public funds. Hence, at the optimal royalty choice,  $\lambda = 1$  from (21) and all required revenues should be collected using the non-distortionary royalty instrument.<sup>10</sup> Moreover, it is not optimal to allocate any of the government's resources to auditing activities. This can be seen by substituting  $\lambda = 1$  in  $\Omega_p^u = 0$ , which gives  $\Omega_p^u|_{\lambda=1} < 0$  so that  $p = 0$ .

Our result is in line with observations given in the literature. Some have argued that in practice many developing country governments are motivated only by royalty revenue generation, and it is often the case that these governments do not enforce illegal logging enough (Johnson 2002). We have shown theoretically that this strategy may indeed be optimal under certain very specific conditions.

When the government responds to amenities this basic result is modified. When  $v'(X^u) < 0$ , there are social costs associated with excess deforestation and illegal logging. The social welfare function is written  $SW = E\pi^* + v(X^u)$ , and we now have the following first order conditions for the Lagrangian function  $\Omega^u = E\pi^* + v(X^u) + \lambda(R^e - \bar{R})$ ,

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<sup>9</sup> This type of policy design problem, along with the specification of the welfare function and the revenue constraint, has been a common feature within the forest taxation design literature in recent years (see Amacher and Brazee 1997, Koskela and Ollikainen 1997).

$$\Omega_t^u = -q\bar{Q} + \lambda q\bar{Q} = 0, \text{ and} \quad (23)$$

$$\Omega_p^u = -fqX^u + v'(X^u)Q_p^u + \lambda[fqX^u + pfqQ_p^u - c'(p)] = 0. \quad (24)$$

From (23) we still find that  $\lambda = 1$  at the optimal policy mix, so that the required revenue need should still be collected using the lump sum royalty instrument. However, there is an additional welfare effect of auditing given by the second term in (24), which represents an increase in amenities (and a reduction in the externality) when the probability of detecting illegal logging increases. It is now optimal for the government to allocate some royalty revenue collections to auditing activities. To see this, assume that the royalty is set at its optimal level,  $t = t^*$ , and then use (23) to rewrite (24) as,

$$\Omega_p^u \Big|_{t=t^*} = pfqQ_p^u - c'(p) + v'(X^u)Q_p^u. \quad (25)$$

Writing this condition at corner solutions of no auditing  $p = 0$  and perfect auditing  $p = 1$  we have  $\Omega_p^u \Big|_{p=0, t=t^*} = v'(X^u)Q_p^u > 0$ , and  $\Omega_p^u \Big|_{p=1, t=t^*} = fqQ_p^u - c'(1) + v'(X^u)Q_p^u < 0$ , respectively. Comparing these suggests it is optimal to devote a share of the government's royalty revenues to auditing efforts.

In order to facilitate interpretation, we re-express (25) as follows,

$$c'(p) = \frac{p}{Q^u} (fq + \frac{1}{p} v'(X^u)) \varepsilon_p^u, \quad (26)$$

where  $\varepsilon_p^u = \frac{Q_p^u p}{Q^u} < 0$  and  $pfq + v'(X^u) < 0$ . Auditing is chosen so that the marginal

cost of auditing (LHS) is equal to the marginal benefit (RHS). The marginal benefit includes the marginal expected fine revenues (first term) plus the marginal benefit of additional amenities that follow from reductions in illegal logging due to auditing (second term). Hence, the amount of resources the government will devote to its auditing strategy depends on the difference between revenue collection changes and

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<sup>10</sup> For a formal discussion of  $\lambda$  and its implications for optimal tax instrument design in general, see Atkinson and Stiglitz (1980), or Myles (1995).

externality reductions that follow from policy-induced reductions in logging. When amenities are important to the government, there is always a benefit to auditing and detection of illegal logging.

### 3.2 Penalty charged on evaded royalty payment

Under this penalty scheme, the royalty rate enters the harvester's penalty payment explicitly. Changing the expected revenue constraint to reflect this, and assuming first that the government omits or ignores the negative externalities from deforestation, we have the following maximization problem:  $Max SW = E\pi^*$  subject to  $R = tq\bar{Q} + ptfqX - c(p) \geq \bar{R}$ . The necessary conditions for the Lagrangian  $\Omega = E\pi^* + \lambda(R - \bar{R})$  in terms of the government's choices of  $t$  and  $p$  are:

$$\Omega_t = -(q\bar{Q} + pqfX) + \lambda[q\bar{Q} + pqfX + pqftQ_t] = 0, \text{ and} \quad (27)$$

$$\Omega_p = -fqtX + \lambda[fqtX + pfqtQ_p - c'(p)] = 0 \quad (28)$$

The marginal revenue effect of the royalty rate clearly differs from the previous penalty scheme. For example, Condition (27) implies  $\lambda > 1$ , so that the marginal cost of public funds now exceeds one. This is due to the fact that, under this penalty scheme, the royalty is distortionary, inducing a negative substitution effect on logging. With regard to the auditing strategy, we infer from (28) that it remains optimal for the government not to allocate resources to detection of illegal logging (notice the second term in brackets on the RHS of (28) is negative under  $\lambda > 1$ ).

Accounting for negative externalities arising from deforestation changes the optimal policy mix once again. The optimal royalty rate and auditing probability are defined by,

$$\Omega_t = -(q\bar{Q} + pqfX) + v'(X)Q_t + \lambda[q\bar{Q} + pqfX + pqftQ_t] = 0, \text{ and} \quad (29)$$

$$\Omega_p = -fqtX + v'(X)Q_p + \lambda[fqtX + pfqtQ_p - c'(p)] = 0. \quad (30)$$

From (29) we can see that  $\lambda > 1$  still holds if the expected revenue loss from the royalty rate ( $\lambda p q f Q_t$ ) is higher than the positive welfare effect due to lower illegal logging ( $v'(X)Q_t$ ). Proceeding as before, we can evaluate (30) assuming  $t = t^*$  at the corner solutions for auditing probabilities to obtain,  $\Omega_p \Big|_{p=0}^{t=t^*} = v'(X)Q_p q \bar{Q} > 0$ , and  $\Omega_p \Big|_{p=1}^{t=t^*} = f q Q_p q \bar{Q} - c'(1)(q \bar{Q} + f q X) + v'(X)Q_p q \bar{Q} < 0$ . These conditions indicate that it is optimal to devote resources to auditing as long as externalities arise from deforestation. We can collect our policy design findings in,

**Result 2. Policy design under risk neutrality.**

*Revenues should always be collected with a positive royalty, which is non-distortionary (distortionary) when the penalty is assessed on undeclared income (evaded royalty payments). While auditing should not be employed to enforce illegal logging in the absence of amenities, it is always optimal in the presence of amenities irrespective of the penalty scheme.*

## 4 Optimal design of royalty scheme and auditing – Risk aversion

Result 1 was derived for a risk neutral harvester. It shows that the penalty scheme does not matter to the choice of policy instruments, and the government should not be expected to enforce illegal logging if it seeks to raise revenue but does not respond to amenities lost from deforestation. We now examine whether these findings are true when harvesters are risk-averse.

The social welfare function is now given by expected indirect utility of the harvester in addition to amenities from protecting forests, i.e.  $SW = EU(\pi)^* + v(X)$ . To further evaluate the design of the royalty instrument, we allow also for the possibility that the royalty is ‘progressive,’ in the sense that the royalty payment includes a lump-sum subsidy denoted by positive  $I$ .<sup>11</sup> This modification is consistent

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<sup>11</sup> Note that progression is simply a change in lump sum income for the harvester. For a discussion of various forms of tax progression, see the classic paper by Musgrave and Thin (1948), and for a further analysis, see chapters 6-8 in Lambert (1993).



with the ways that many royalties are applied in practice where concessions are important.<sup>12</sup>

In the presence of the subsidy, the concessionaire's revenues are  $Y = [qQ - (tq\bar{Q} - I) - c(Q)]$  and  $Z = [Y - fqX]$  when a penalty is assessed against evaded royalty payments, and  $Y^u = [qQ^u - (tq\bar{Q} - I) - c(Q^u)]$  and  $Z^u = [Y^u - fqX]$  for the case of a penalty assessed against unreported income. Using these definitions, expected government royalty collections become  $R^e = (tq\bar{Q} - I) + pfqX^u - c(p)$  and  $R = (tq\bar{Q} - I) + ptfqX - c(p)$  respectively under the two penalty schemes.

#### 4.1 Penalty charged on undeclared income

The government now chooses the royalty rate, the subsidy level, and the audit probability to solve the following problem,

$$\underset{t, I, p}{\text{Max}} SW = EU(\pi)^* + v(X^u) \text{ s.t. } R = (tq\bar{Q} - I) + pfqX^u - c(p) \geq \bar{R}. \quad (31)$$

Like in the previous section, we first study the case where either there is no externality associated with deforestation or the government ignores it, and then we consider the case where amenities are important.

##### A. Amenities Ignored

With the amenity term  $v(X^u)$  set equal to zero, the first order conditions for the problem in (31) for the royalty rate, subsidy and auditing are written,

$$\Omega_t^u = -[(1-p)u'(Y^u) + pu'(Z^u)]q\bar{Q} + \lambda[q\bar{Q} + pfqQ_t^u] = 0, \quad (32)$$

$$\Omega_I^u = [(1-p)u'(Y^u) + pu'(Z^u)] - \lambda[1 - pfqQ_t^u] = 0, \text{ and} \quad (33)$$

$$\Omega_p^u = -[u(Y^u) + u(Z^u)] + \lambda[fqX^u + pfqQ_p^u - c'(p)] = 0. \quad (34)$$

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<sup>12</sup> This royalty structure is equivalent to instruments used in many tropical forest countries, including Indonesia, The Philippines, and parts of Latin America (Gray 2000, Vincent 1990, Amacher et al. 2001).

According to (32) the government chooses the instrument to equate marginal revenue collections (RHS) with the marginal effect of the royalty on welfare, measured in expected utility terms (LHS). Using the fact that  $Q_t'' = -q\bar{Q}Q_I''$  in (32) and (33), it is easy to re-express the optimality condition for  $I$  in a form identical to the optimal royalty choice. Doing so shows that a subsidy is not needed when the royalty rate is set at its optimal level.<sup>13</sup> There is thus no need for royalty progression as long as the penalty from illegal logging is levied on undeclared income. We show later that this result will change when the penalty scheme is different.

Under this penalty scheme we can express the optimal royalty rate in closed form, and investigate how it depends on parameters of the problem. From the first-order conditions we have,

$$\Omega_t'' = 0 \Leftrightarrow pu'(Z'') \frac{fq}{q - c'(Q'')} = \lambda \left[ 1 + \frac{pfQ''}{\bar{Q}} \varepsilon_t \right], \text{ where } \varepsilon_t = \frac{Q_t''}{Q''} \leq 0 \text{ as } A' \leq 0. \text{ Solving}$$

for  $Z''$  gives  $Z'' = u'^{-1}(\Delta)$ , with  $\Delta = (1 + \frac{pfQ''}{\bar{Q}} \varepsilon_t)$ . Using next the definition of

$Z''$  yields as the optimal royalty rate:

$$t^* = \frac{qQ'' - c(Q'') - fqX''}{q\bar{Q}} - \frac{u'^{-1}(\Delta)}{q\bar{Q}}. \quad (35)$$

Notice first that the derivative of the optimal royalty rate with respect to  $\Delta$  is

$$t_{\Delta}^* = -\frac{u''^{-1}(\Delta)}{q\bar{Q}} > 0. \text{ Thus, we can examine how the optimal royalty rate depends on the}$$

other parameters of the model. The optimal royalty rate  $t^*$  has the following properties:

$$t_{\varepsilon_t}^* < 0, t_p^* < 0, \text{ and } t_f^* < 0.$$

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The lump sum fee is consistent with the many subsidies given to concessionaires by developing country governments, such as road construction or logging equipment cost sharing.

<sup>13</sup> The same happens if  $I$  is a lump-sum concessions fee paid by the harvester. In this case we can rewrite (33) as follows  $\Omega_I'' = -[(1-p)u'(Y'') + pu'(Z'')] + \lambda[1 + pfqQ_I''] = 0$ , where  $Q_I'' = q\bar{Q}Q_I''$  indicating that (32) and (33) are equal. See also Appendix 2.

There are three interpretations that follow from (25). First, the optimal royalty rate decreases as the elasticity of harvesting with respect to the royalty rate increases. This is known as an ‘inverse elasticity rule’ in the optimal taxation literature (see, e.g. Atkinson and Stiglitz 1980). When harvests are very elastic in terms the royalty rate, large royalty rates reduce welfare more and increase the inability of the government to collect revenues as harvesting declines. Thus, the royalty rate would be set at a lower level. The opposite is true if harvesting elasticity is low. Second, the audit probability and the penalty rate are substitute instruments for the royalty rate. If the probability of detection is low, as it is in most developing countries, then higher royalty rates are needed for the government to satisfy its revenue constraint, *ceteris paribus*; this is because illegal logging, which reduces revenues the government captures, increases as the probability of detection decreases. Third, the optimal royalty rate is lower when the fine imposed for illegal logging is higher. These three results collectively show that the design of royalty instruments and of auditing and fines cannot be undertaken separately, as they are often discussed in the literature on concessions and illegal logging.

Finally, we can also show how resources should be allocated for detection of illegal logging. From (34) we develop the following modified condition,

$$c'(p) = \frac{u(Z^u) - u(Y^u)}{\lambda} + fqX^u \left(1 + \frac{Q^u}{X^u} \varepsilon_p\right). \quad (36)$$

From (36), it is *always* optimal for the government to devote resources to auditing. The allocation of resources to detection activities depends on the direct effect on the expected utility of these efforts, and on the sum of the immediate and indirect effects of higher detection rate on illegal logging as well as on the marginal cost of public funds.

We summarize our findings in,

**Proposition 1. Optimal royalty and auditing design when penalty is levied on undeclared income.**

*When amenities are not present (or are ignored by the government) and given the optimal royalty rate is used, introducing a lump-sum subsidy or a lump-sum fee is not optimal. The optimal royalty rate is inversely related to the elasticity*

*of harvesting with respect to the royalty, the audit probability, and the fine rate. Auditing of illegal logging is always optimal.*

It is worth noting that most developing countries using royalties almost always use a subsidy for harvesters.

## B. Amenities included

When amenities are present and the government responds to them, the first-order conditions for the government's choice of instruments become,

$$\Omega_t'' = -[(1-p)u'(Y'') + pu'(Z'')]q\bar{Q} + v'(X'')Q_t'' + \lambda[q\bar{Q} + pfqQ_t''] = 0, \quad (37)$$

$$\Omega_t'' = [(1-p)u'(Y'') + pu'(Z'')] + v'(X'')Q_t'' - \lambda[1 - pfqQ_t''] = 0, \text{ and} \quad (38)$$

$$\Omega_p'' = -[u'(Y'') - u(Z'')] + v'(X'')Q_p'' + \lambda[fqX'' + pfqQ_p'' - c'(p)] = 0, \quad (39)$$

where (38) describes the optimal lump-sum subsidy. As is evident from (37) and (38), our earlier results remain valid. The same holds also in the case of a lump sum royalty fee. That is, it is never optimal for the government to employ the lump-sum subsidy or fee given the optimal royalty rate as long as the penalty is levied on the undeclared income. For the optimal auditing policy, we can after some rearranging arrive at the following condition,

$$c'(p) = \frac{u(Z'') - u(Y'')}{\lambda} + \frac{v'(X'')}{\lambda} \frac{q}{Q''} \varepsilon_p + fqX''(1 + \frac{Q''}{X''} \varepsilon_p) \quad (40)$$

The LHS is the marginal cost of auditing for a given probability of detection. the RHS measures the welfare cost of the policy on harvesters (first term), the welfare benefit of reductions in deforestation (second term), and marginal revenue collection by the government (third term). Notice that, relative to the case where negative externalities were absent, the RHS of this condition is larger. Thus, given the convex auditing cost function, it is optimal for the society to devote more resources to auditing activities in this case, as the marginal benefits of reductions in illegal logging are greater. This

implies, naturally, that under DARA the royalty rate will also be higher than in the previous case.

**Corollary 1. Importance of amenities when penalty is levied on undeclared income**

*When negative externalities arise from illegal logging, then given the optimal royalty rate, introducing a lump-sum royalty subsidy or a lump-sum fee is not optimal. The optimal royalty rate is higher under decreasing absolute risk aversion, and auditing is higher when amenities are present.*

**4.2 Penalty charged on evaded royalty payment**

When the penalty is charged against evaded royalty payments the concessionaire's revenues for the alternative outcomes under non-detection and detection of illegal logging are,  $Y = [qQ - (tq\bar{Q} - I) - c(Q)]$  and  $Z = [Y - tfqX]$ , respectively. The government's revenue constraint is also modified accordingly, and the policy choice problem becomes,

$$\underset{t, I, p}{\text{Max}} SW = EU(\pi)^* + v(X) \quad \text{s.t.} \quad R = (tq\bar{Q} - I) + ptfqX - c(p) \geq \bar{R} \quad (41)$$

**A. Amenities Ignored**

Neglecting for the moment the externality term, the first order conditions are,

$$\Omega_t = -[(1-p)u'(Y)q\bar{Q} + pu'(Z)(q\bar{Q} + qfX)] + \lambda[(q\bar{Q} + pqfX) + pfqtQ_t] = 0, \quad (42)$$

$$\Omega_I = [(1-p)u'(Y) + pu'(Z)] + \lambda[-1 + pfqtQ_I] = 0, \quad \text{and} \quad (43)$$

$$\Omega_p = -[u(Y) - u(Z)] + \lambda[tfqX + pfqtQ_p - c'(p)] = 0, \quad (44)$$

where (43) describes the optimal lump-sum subsidy. Using the condition  $\Omega_I = 0$ , we

have  $\lambda = \frac{(1-p)u'(Y) + pu'(Z)}{1 - pfqtQ_I} \geq 1$ , so that  $\lambda \geq (1-p)u'(Y) + pu'(Z)$  as  $Q_I \geq 0$ . Given

$I = I^*$  the first-order condition (42) can be re-expressed after some rearranging as,

$$t^* : \frac{u'(Z) - \lambda}{\lambda} = \frac{t}{X} [q\bar{Q}Q_I + Q_I], \quad (42')$$

where  $[q\bar{Q}Q_I + Q_I] = Q_I^c - xq\bar{Q}Q_I < 0$  (see Appendix 2). Since the LHS of (42') is negative, the optimal royalty rate  $t^*$  is positive. According to (43), the optimal subsidy should be chosen in a manner that equates marginal welfare gain to the concessionaire,  $(1-p)u'(Y) + pu'(Z)$ , to marginal cost of public funds due to royalty revenue collections,  $\lambda[1 - pfqtQ_I]$ . If we have the lump-sum fee, instead of a subsidy, then we can rewrite (43) as follows  $\Omega_I = -[(1-p)u'(Y) + pu'(Z)] + \lambda[1 + pfqtQ_I] = 0$ , where  $Q_I \leq 0$  as  $A'(\cdot) \leq 0$ . In this case the Slutsky equation for the royalty rate  $t$  is  $Q_I = Q_I^c + q\bar{Q}(1-x)Q_I$ . Given  $I = I^*$  the first-order condition (42) can now be re-expressed to obtain,

$$t^* : \frac{u'(Z) - \lambda}{\lambda} = \frac{t}{X} [-q\bar{Q}Q_I + Q_I], \quad (42'')$$

where  $-[q\bar{Q}Q_I + Q_I] = Q_I^c + xq\bar{Q}Q_I < 0$ . Hence, the optimal royalty rate  $t^*$  is now positive, so that when penalties are charged against evaded royalties, both tax instruments (the royalty rate and the lump-sum subsidy/lump-sum fee) should be used – this is different than the policy mix found under risk neutrality. This raises the following question: should the government use a subsidy or a fee as a supplement to the royalty rate? The answer depends on the size of the tax revenue requirement the government seeks to collect. If the royalty rate applied together with the auditing strategy is not sufficient to collect the revenue requirement, then the government should use a lump-sum fee, and not the subsidy, when using a royalty. This is counter to the typical practice of using both a royalty and a subsidy by governments with concessions in tropical countries. In the case we have established, this means the optimal royalty structure is *regressive*.

Finally we turn to the government's auditing strategy. From the corresponding condition for  $\Omega_p = 0$  in (44), we can establish that it is *always* optimal to devote

resources to auditing for illegal logging. The allocation of resources is defined by the following condition,

$$c'(p) = \frac{u(Z) - u(Y)}{\lambda} + fqX(1 + \frac{Q}{X}\varepsilon_p) \quad (45)$$

Thus, the allocation of auditing resources depends on its negative direct effect on the expected utility of the concessionaire, as well as the positive sum of immediate and indirect effects of higher detection rates on illegal logging and, thus, government revenue collections. The following proposition summarizes the above results,

**Proposition 2. Optimal royalty and auditing design when penalty is levied on evaded royalty payments**

*When amenities are not present (or are ignored by the government), the optimal royalty rate is positive and the royalty system may be progressive or regressive. The higher the government tax revenue requirement, the more likely the system should be regressive. Auditing of illegal logging is always optimal.*

**B. Amenities included**

Finally, we consider briefly how externalities modify policy choices under this payment scheme. Now the first-order conditions governing the choice of instruments under tax exemption are,

$$\Omega_t = -[(1-p)u'(Y)q\bar{Q} + pu'(Z)(q\bar{Q} + qfX)] + v'(X)Q_t + \lambda[(q\bar{Q} + pqfX) + pfqtQ_t] = 0 \quad (46)$$

$$\Omega_l = [(1-p)u'(Y) + pu'(Z)] + v'(X)Q_l - \lambda[1 - pfqtQ_l] = 0, \text{ and} \quad (47)$$

$$\Omega_p = -[u(Y) - u(Z)] + v'(X)Q_p + \lambda[fqtX + pfqtQ_p - c'(p)] = 0. \quad (48)$$

As is evident from the conditions  $\Omega_t = 0$  and  $\Omega_l = 0$ , our previous results continue to hold, when externalities arise through deforestation, if the expected revenue loss from a higher royalty rate ( $\lambda pqfQ_t$ ) is greater than the positive welfare effect due to resulting lower illegal logging ( $v'(X)Q_t$ ). Looking at the second RHS terms in (46-48) we can see that under this condition the royalty rate is higher and the subsidy lower than in the absence of amenities. This implies that allowing for amenities will increase the

likelihood of a regressive royalty tax scheme being optimal, because higher royalty rates and higher lump sum fees will decrease illegal logging.

For the optimal probability of detection we have,

$$c'(p) = \frac{u(Z) - u(Y)}{\lambda} + \frac{v'(X)}{\lambda} \frac{q}{Q} \varepsilon_p + fqX \left(1 + \frac{Q}{X} \varepsilon_p\right), \quad (49)$$

Thus, the RHS of (49) is higher than the RHS of (45), and so the government should devote more resources to auditing when externalities are present given that enforcement costs are convex. This finding clearly indicates that it is optimal for the society to devote more resources to auditing activities in order to prevent illegal logging and deforestation. We summarize these findings as,

**Corollary 2. Importance of amenities to policy design when penalty is levied on evaded royalty payments.**

*When amenities are present, the optimal royalty rate is positive and positive auditing effort continues to be optimal, but more resources should be devoted to auditing compared to the case where amenities are absent. The optimal royalty rate is also higher, while the optimal lump-sum subsidy is lower or equivalently the optimal lump sum fee is higher.*

## 5. Conclusions

We have examined royalty and auditing choice in a context where concessions are allocated by the government and illegal logging can occur. The government collects a royalty fee applied to the concession – the type of royalty we consider here is common to many tropical forest situations. The form of illegal logging examined is also the most common, i.e., where a concessionaire harvests more than is contracted for by the concession. We also assume, realistically, that illegal logging activity is not detectable by the government unless costly enforcement is employed. Enforcement proceeds through auditing, which ensures that detection occurs with some positive probability. Should the government detect illegal logging, two possible penalty schemes are modeled. These include a penalty levied on undeclared harvest income, and a penalty levied on evaded royalty payments. Our assumptions of probabilistic detection and



costs of auditing are consistent with the fact that in tropical forest countries enforcement might be difficult to conduct.

Within this framework we evaluate two questions: First, how incentives for harvesters to illegally log depend on government auditing effort and royalty choices and second, how a government should choose royalties and the audit frequency when facing a revenue collection need. Increasing auditing probabilities is consistent with increasing the probability of detection of illegal logging, as more government resources are devoted to enforcement. We allow for the realistic notion of a government interested in generating royalty revenue collections through concessions, potentially interested in the amenities reduced by illegal excessive logging, and also interested in the welfare of concessionaires.

Our results provide some closure to the current debate of how to control illegal logging through policy choices, even when the government is highly revenue-motivated. Most importantly, we establish how royalties and enforcement strategies must be chosen jointly when illegal logging confounds both control deforestation and ensure government revenue collections. The objectives of a government acting as a social planner, captured through the definition of net social benefits, are important in determining what policy mix can and should be employed. We also provide some intuition about why enforcement is sometimes not undertaken by governments.

When harvesters are risk neutral, policy choices depend on the presence of amenities, but not on the penalty scheme. Given the realistic conditions under which we examine royalties, the government will never find it optimal to completely eliminate illegal logging even when forest amenities are important, at least when auditing is costly and the fine is given. When amenities are not present, the royalty should be employed to collect all revenues, while auditing should not be employed to enforce illegal logging. When amenities are present, both royalties and auditing of illegal logging should be used.

When harvesters are risk-averse the results are different. In the absence of amenities and when penalties are assessed on undeclared income, neither a lump-sum subsidy nor a lump-sum fee assessed for the concession are needed, while the optimal royalty rate is inversely related to the elasticity of harvesting with respect to the royalty, the audit probability, and the fine rate. Auditing of illegal logging is also optimal.

Finally, when amenities are ignored and penalties are levied on evaded royalty payments, the royalty system may be progressive or regressive depending on the size of the government's revenue collection requirement. When amenities are present the royalty rate should be higher, progression lower, and regression higher. Also, more resources should be devoted to auditing compared to the case where amenities are absent.

What does our analysis say about the current debate and application of royalties to control illegal logging and still allow governments adequate royalty revenues? First, our results show how governments should balance the effects of royalties and enforcement on illegal logging, in order to generate revenues and provide the right level of public goods. Royalties and detection of illegal logging must be considered as a system and not as separate instruments (in the literature royalties are nearly always considered without regards to illegal logging or enforcement efficiency). The risk preferences of harvesters are also important in determining their incentive to capture rents in risky manners, and policy design must account for this. The type of penalty system chosen, although it has not been discussed in the literature, can be important in predicting both the impact and choice of royalty and auditing instruments. Finally, and interestingly, we establish cases where a government would not choose to audit for illegal logging, i.e., when harvesters are risk neutral and the government does not respond to externalities associated with deforestation or illegal logging. We also show, contrary to what many people and organizations recommend, that a government in practice choosing royalties under conditions we examine would never find it optimal to drive illegal logging to zero.

## Appendix 1. Comparative Statics of Instruments under Risk-Aversion

Penalty fine rate, and detection probability:

$$Q_f^u = -\frac{EU_{Q^u f}}{EU_{Q^u Q^u}} < 0, \text{ where } EU_{Q^u f} = -[qX^u u''(Z^u)b + qu'(Z^u)] < 0 \quad A1.1$$

$$Q_p^u = -\frac{EU_{Q^u p}}{EU_{Q^u Q^u}} < 0, \text{ where } EU_{Q^u p} = -u'(Y^u)a + u'(Z^u)b < 0. \quad A1.2$$

Exogenous income and royalty rate:

$$Q_I^u = -\frac{EU_{Q^u I}}{EU_{Q^u Q^u}} \leq 0, \text{ when } A'(\cdot) \leq 0, \quad A1.3$$

where  $EU_{Q^u I} = -a(1-p)u'(Y^u)[A(Y^u) - A(Z^u)] \geq 0$ , and

$$Q_t^u = -\frac{EU_{Q^u t}}{EU_{Q^u Q^u}} \leq 0, \text{ when } A'(\cdot) \leq 0, \quad A1.4$$

where  $EU_{Q_t^u} = q\bar{Q}a(1-p)u'(Y^u)[A(Y^u) - A(Z^u)] \leq 0$ . A1.4 can be decomposed into the income and substitution effects (see Appendix 2)

$$Q_t^u = Q_t^c - q\bar{Q}Q_I^u, \quad \text{where} \quad Q_t^c = Q_t^u + g\bar{Q}Q_I^u = 0 \quad A1.5$$

so that the substitution effect of the royalty rate is zero.

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## Appendix 2. Slutsky Decompositions for the Royalty Rate

We develop Slutsky decompositions for the total effect of the royalty rate  $t$  on logging under the alternative penalty schemes.

### A. Penalty on undeclared income

The concessionaire's expected utility is,

$$EU(\pi) = (1-p)u(Y^u) + pu(Z^u) \quad A2.1$$

where  $Y^u = qQ^u - c(Q^u) - tq\bar{Q} + I$ ,  $Z^u = Y^u - fq(Q^u - \bar{Q})$  and  $I$  is the non-logging income or lump-sum subsidy. The first-order condition for the optimal logging  $EU_{Q^u} = 0$  implicitly defines  $Q^u$  as  $Q^u \equiv Q^u(t, I)$ . Substituting  $Q^u(t, I)$  for  $Q$  in A.2.1 gives expected indirect utility  $EU^*(t, I) = U^o$ , with the following properties holding due to the envelope theorem,

$$EU_I^* = (1-p)u'(Y^u) + pu'(Z^u) > 0, \text{ and} \quad \text{A2.2a}$$

$$EU_t^* = -q\bar{Q}EU_I^* < 0. \quad \text{A2.2b}$$

Inverting  $EU^*$  for  $I$  to obtain  $I = h(t, U^o)$  and substituting this for  $I$  in  $EU^*$  we obtain the compensated expected indirect utility function (see Diamond-Yaari (1972))

$$EU^*(t, h(t, U^o)) = U^o \quad \text{A2.3}$$

Differentiating A2.3 with respect to the royalty tax rate yields:  $EU_t^* + EU_I^*h_t = 0$ , so that we have the following relationship between the royalty and the non-logging income  $I$ , holding the expected utility constant,

$$h_t = -\frac{EU_t^*}{EU_I^*} = q\bar{Q} > 0 \quad \text{A2.4}$$

From the duality theorem (see e.g. Varian 1992, pp. 81-93), we can write the relationship between uncompensated logging,  $Q^u$ , and compensated logging,  $Q^c$ , as follows,

$$Q^u(t, h(t, U^o)) = Q^c(t, U^o) \quad \text{A2.5}$$

Differentiating A2.5 with respect to the royalty rate yields  $Q_t + Q_I h_t = Q_t^c$ . Using A2.4 we arrive at the Slutsky equation for the royalty rate,

$$Q_t^u = Q_t^c - q\bar{Q}Q_I^u, \quad \text{A2.6}$$

where the total effect is decomposed into the substitution effect ( $Q_t^c$ ) and the income effect ( $-q\bar{Q}Q_I^u$ ).

We know from our discussion in section 2.1 that the total effect of the royalty rate is given by the expression,

$$Q_t^u = -\frac{EU_{Q^u t}}{EU_{Q^u Q^u}} 0, \text{ where } EU_{Q^u t} = q\bar{Q}(q - c'(Q^u))(1-p)u'(Y^u)[A(Y^u) - A(Z^u)]$$

The effect of non-logging income can be expressed similarly as,

$$Q_I^u = -\frac{EU_{Q^u I}}{EU_{Q^u Q^u}}, \text{ where } EU_{Q^u I} = -(q - c'(Q^u))(1-p)u'(Y^u)[A(Y^u) - A(Z^u)]$$

Thus in the case of the penalty assessed on undeclared income we have,

$$Q_t^c = Q_t^u + q\bar{Q}Q_I^u = 0. \quad \text{A2.7}$$

B. Penalty on evaded royalty payment

When the penalty is assessed on evaded royalty payments, the concessionaire's expected utility income terms in A.1 are written  $Y = qQ - c(Q) - tq\bar{Q} + I$  and  $Z = Y - ftq(Q - \bar{Q})$ . Proceeding as above we can derive the following properties of the expected indirect utility function in terms of the royalty rate and the non-logging income

$$EU_I^* = (1 - p)u'(Y) + pu'(Z) > 0 \quad \text{A2.8a}$$

$$EU_t^* = -q\bar{Q}(1 + x)EU_I^* > 0 \quad \text{A2.8b}$$

where  $x = \frac{fX}{\bar{Q}} > 0$ . Like in the earlier case we can use A2.8a to invert the expected indirect utility function and obtain the corresponding compensated utility function, which now yields the following relationship between the royalty rate and the non-logging income, holding expected utility constant

$$h_I = -\frac{EU_t^*}{EU_I^*} = q\bar{Q}(1 + x) > 0 \quad \text{A2.9}$$

Using the relationship between uncompensated and compensated logging behavior, like in A2.5, and equation A2.9, we can derive the following Slutsky equation for the effect of the royalty rate on logging,

$$Q_t = Q_t^c - q\bar{Q}(1 + x)Q_I. \quad \text{A2.10}$$

We know from the discussion in the text that the total effect of the royalty rate is given by,

$$Q_t = -\frac{EU_{Q_t}}{EU_{QQ}} < 0 \quad \text{A2.11}$$

where  $EU_{Q_t} = q\bar{Q}(1 + x)(q - c'(Q))(1 - p)u'(Y)[A(Y) - A(Z)] - pu'(Z)fq < 0$  under non-increasing absolute risk aversion. Moreover, we have previously shown,

$$Q_I = -\frac{EU_{QI}}{EU_{QQ}} \text{ where } EU_{QI} = -(q - c'(Q))(1 - p)u'(Y)[A(Y) - A(Z)] \geq 0 \text{ as } A'(\cdot) \leq 0 \quad \text{A2.12}$$

Using A2.11 and A2.12 yields,

$$Q_t^c = Q_t + q\bar{Q}(1 + x)Q_I = (-EU_{QQ})^{-1}\{-fqpu'(Z)\} < 0, \quad \text{A2.13}$$

Hence, when the penalty is assessed on the evaded royalty payment, and risk aversion is non-increasing, the substitution effect of the royalty rate on logging is negative.

In the case of lump sum tax – i.e. when  $I < 0$  – we can derive the following Slutsky equation for the effect of the royalty rate on logging,

$$Q_t = Q_t^c + q\bar{Q}(1 + x)Q_I. \quad \text{A2.10'}$$

where  $Q_t < 0$  and  $Q_I \leq 0$  as  $A'(\cdot) \leq 0$ . Now we end up with

$$Q_t^c = Q_t - q\bar{Q}(1+x)Q_I = (-EU_{QQ})^{-1}\{-fqp u'(Z)\} < 0, \quad \text{A2.13'}$$

like in the earlier case.

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